ENTROPY LOSSES IN THE FLOW OF A DUST-LADEN GAS SUSPENSION IN A NOZZLE

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The unidimensional steady-state flow of a gas containing a system of solid particles in a nozzle is considered for the case in which the particles are characterized by a constant velocity lag; expressions are derived for the entropy losses in efficiency due to the friction between the gas and the particles and the irreversibility of interphase heat transfer.

The entropy losses which take place in a gas flow containing solid particles are due to viscous dissipation in the gas itself, the friction of individual particles in the boundary layers, the friction of the gas and particles on the wall of the channel, and the irreversibility of interphase heat-transfer processes and heat transfer between the flow and the channel walls. The frictional losses of the gas in relation to the particles and channel walls may be regarded as (approximately) additive, since most of the particles are concentrated close to the axis of the nozzle [1].

Let us consider the entropy losses due to the friction between the gas and the particles ΔE_1 and the irreversibility of the interphase heat transfer ΔE_2. In accordance with [2] we may write

$$\Delta E = \Delta E_1 + \Delta E_2 = T_0 \int_0^\tau \dot{s}^{(1)}\, dt + T_0 \int_0^\tau \dot{s}^{(2)}\, dt. \tag{1}$$

In order to find the entropy derivatives $s^{(1)}$ and $s^{(2)}$ we use the model of the unidimensional steady-state flow of a gas containing solid particles in a nozzle (with a constant velocity lag of the particles) [3], and also the corresponding analytical solutions, which agree to within 5% with the results of more rigorous computer calculations based on variable lag and existing experimental data. Since the model considered in [3] makes no allowance for dissipation in the actual gas or heat transfer to the channel walls, the losses due to these processes fail to appear in ΔE_1 and ΔE_2. For fine particles we may put $T_s - T \ll T$. The quantity $s^{(1)}$ may be expressed in terms of a dissipative function, which takes the following form for the model under consideration [4]

$$T\dot{s}^{(1)} = \sum_{j=1}^{N} \left[B_j (\mathbf{u} - \mathbf{u}_s) \nabla^2 \mathbf{u} + G_j (\mathbf{u} - \mathbf{u}_s)^2 \right], \tag{2}$$

B_j and G_j are proportionality factors, and the summation extends over all the particles in the system. Flow with a constant lag $k = (u - u_s)/u$ is characterized by a gas-velocity gradient which is constant along the whole channel [3]

$$\frac{du}{dx} = A = \frac{9}{2} \frac{\eta}{\rho_s r^2} \frac{k}{(1-k)^2}. \tag{3}$$

Here the first term in the dissipative function (2) vanishes. For a system of N_0 identical spherical particles in unit mass of gas we may write

$$T\dot{s}^{(1)} = G N_0 u^2 k^2, \tag{4}$$

where

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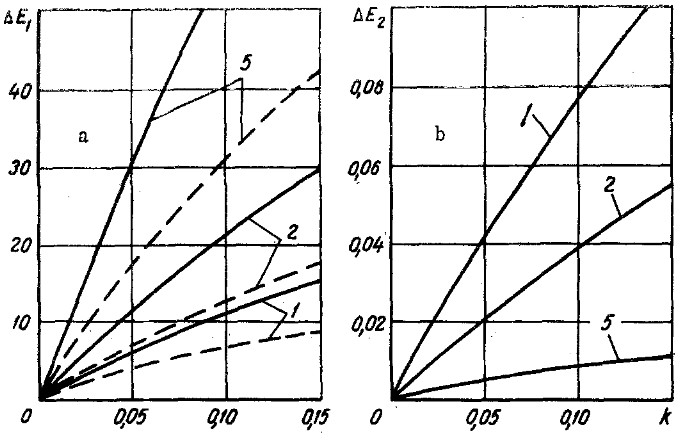

Fig. 1

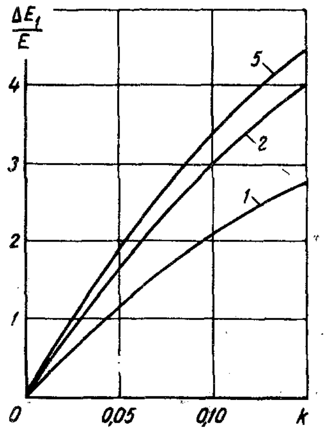

Fig. 2

Fig. 1. Entropy losses associated with friction (a) and with the irreversibility of interphase heat transfer (b) (for the continuous curves $p_1/p_2 = 5$, for the broken curves 2.5; the figures on the curves give the values of μ); ΔE_1, ΔE_2 in kJ/kg.

Fig. 2. Relative losses due to friction for $p_1/p_2 = 5$ (figures on the curves give the values of μ); $\Delta E_1/E$ in %.

$$N_0 = \frac{\mu}{\rho_s}\,\frac{3}{4\pi r^3}\,. \tag{5}$$

From (4) and (5)

$$\dot{s}^{(1)} = \frac{3\mu G u^2 k^2}{4\pi\rho_s r^3 T}\,. \tag{6}$$

The law of temperature variation along the nozzle may be found from the energy equation for a polytropic gas flow

$$\frac{d}{dx}\left(\frac{u^2}{2}\right) = -\frac{nR}{n-1}\,\frac{dT}{dx}\,. \tag{7}$$

Integrating (7) with due allowance for (3) and neglecting the velocities of the gas and particles at the entrance into the nozzle, we have

$$T = T_1 - cx^2, \tag{8}$$

$$c = \frac{A^2(n-1)}{2Rn}\,. \tag{9}$$

Let us transform to the new variable $dt = dx/v$ in (1) by introducing the center-of-mass velocity

$$v = \frac{\mu(1-k)+1}{\mu+1}\,u. \tag{10}$$

The entropy losses ΔE_1 referred to unit mass of gas may be found from (1) allowing for (3), (6), (8), (9), and (10):

$$\Delta E_1 = \frac{3T_0 G\mu(\mu+1)k^2 A}{4\pi\rho_s r^3[\mu(1-k)+1]}\int_0^h \frac{x\,dx}{T_1 - cx^2}\,. \tag{11}$$

In accordance with [1, 3-6], the quantity G may be expressed as

$$G = 6\pi\eta r. \tag{12}$$

Carrying out the integration in (11) and allowing for the explicit form of G from (12) and A from (3), we obtain

$$\Delta E_1 = \frac{\mu(\mu+1)k(1-k)^2}{[\mu(1-k)+1]}\,\frac{n}{n-1}\,RT_0\ln\frac{T_1}{T_2}\,, \tag{13}$$

where

$$T_2 \equiv T(h) = T_1 - ch^2. \tag{14}$$

For calculating ΔE_2 we make use of the results of [7]

$$\Delta E_2 = T_0 \int_0^\tau \dot{s}^{(2)} dt = \frac{3\mu T_0 \alpha}{r\rho_s} \int_0^\tau \left(\frac{T_s - T}{T}\right)^2 dt. \tag{15}$$

A constant velocity lag leads to a constant temperature lag L [3]

$$L = \frac{T_1 - T_s}{T_1 - T}. \tag{16}$$

From (7) and (16) we obtain

$$\frac{T_s - T}{T} = \frac{(1 - L)(n - 1)}{2nR} \frac{u^2}{T}. \tag{17}$$

If in (15) we transform to a new variable $dt = dx/v$ and allow for (3), (8), (9), (10), and (17), we obtain

$$\Delta E_2 = \frac{3\mu(\mu + 1) T_0 \alpha (1 - L)^2 (n - 1)^2 A^3}{[\mu(1 - k) + 1] r\rho_s (2nR)^2} \int_0^h \frac{x^3 dx}{[T_1 - cx^2]^2}. \tag{18}$$

Let us make use of the criteria (numbers)

$$\mathrm{Nu} = \frac{2\alpha_0 r}{\lambda}; \ \mathrm{Bi} = \frac{\alpha_0 r}{\lambda_s}; \ \mathrm{Pr} = \frac{\nu}{a}. \tag{19}$$

The effective heat-transfer coefficient may be written in the following way, allowing for the internal thermal resistance of the particles [8]:

$$\alpha = \frac{\alpha_0}{1 + \varphi \mathrm{Bi}}. \tag{20}$$

The quantity φ depends on the shape of the body, but only slightly on Bi. Carrying out the integration in (18) and remembering (3), (9), (19), and (20), we obtain

$$\Delta E_2 = \frac{1}{6} c_p T_0 \frac{\mathrm{Nu}}{(1 + \varphi \mathrm{Bi}) \mathrm{Pr}} \frac{\mu(\mu + 1)}{[\mu(1 - k) + 1]} \frac{(1 - L)^2 (1 - k)^2}{k} \left(\frac{T_1 - T_2}{T_2} - \ln \frac{T_1}{T_2}\right). \tag{21}$$

Figure 1 shows the values of ΔE_1 and ΔE_2 calculated from (13) and (21) for an air–graphite suspension. In the calculations we used $c_p \approx c_s = 1$ kJ/kg·deg, $R = 287$ J/kg·deg, $T_0 = 300$°K, $\varphi \mathrm{Bi} \ll 1$; n, L, Nu were calculated in accordance with [3].

Figure 2 shows the relative entropy losses due to the velocity lag of the particles. In calculating the efficiency E in front of the nozzle we took $T_1 = 800$°K. The ratio T_1/T_2 was calculated from the polytropic equation for the specified pressure ratio p_1/p_2.

NOTATION

t	is the time;
T_0	is the temperature of the surrounding medium;
T, T_s	are the gas and particle temperatures;
u, u_s	are the gas and particle velocities;
A	is the gas-velocity gradient;
x	is the axial coordinate;
η	is the dynamic viscosity of the gas;
ρ_s	is the density of the particle material;
N_0	is the number of particles in unit mass of gas;
r	is the radius of particle;
μ	is the mass concentration of particles;
n	is the polytropic index;

R	is the gas constant;
T_1, T_2	are the gas temperatures at entrance and exit of the nozzle;
h	is the length of nozzle;
λ, λ_s	are the thermal conductivities of gas and particles;
c_p, c_s	are the specific heats of gas and particles;
α	is the heat-transfer coefficient of particles and gas;
ν, a	are the kinematic viscosity and thermal diffusivity of gas;
p_1, p_2	are the gas pressures in front of and behind the nozzle.

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